Challenge Number 2: Sounds Pretty Radical, Pt. I

THE MATH JESTER

Fall 2021

Name: _____ Date: _____

Directions

Complete as many of the following problems as you can. You may use a calculator or Unix terminal for help with calculations, but show your work! Partial credit will be awarded for good reasoning.

1. Consider $\sqrt{612}$. Your mission, should you choose to accept it, is to convert this expression into $\sqrt{M^2N}$, where M and N are integers. What is the largest value of M that satisfies this equation? What is the corresponding value of N? [2 points]
   Hint: if you do this correctly, N will be an example of a *squarefree* [1] factor of 612. What does this mean? How can you apply it?

2. Now consider $\sqrt[3]{17000}$. Convert *this* expression into $\sqrt[3]{M^3N}$, where M and N are integers. What is the largest value of M that satisfies this equation? What is its corresponding value of N? [4 points]
   Hint: If you do this correctly, N is a *cubefree*[2] factor of 17000. What does this tell you?

3. Consider $\sqrt[5]{9143008}$. Convert this expression into $\sqrt[5]{M^5N}$, where M and N are integers. What is the largest value of M that satisfies this equation? What is its corresponding value of N? [4 points]

4. Now consider $\sqrt[6]{1927458368}$. Then use the strategies or patterns presented above in order to show that this expression is equal to $\sqrt[3]{\sqrt[2]{1927458368}}$. Why do you think this is? Can you generalize this fact to a number other than 1927458368? [5 points]

5. Is this problem interesting? Why or why not? [3 points]

[1] Weisstein, Eric W. "Squarefree." From MathWorld – A Wolfram Web Resource. https://mathworld.wolfram.com/Squarefree.html

[2] —. "Cubefree." From MathWorld – A Wolfram Web Resource. https://mathworld.wolfram.com/Cubefree.html