Challenge Number 3: Sounds Pretty Rational, Pt. I

THE MATH JESTER

Fall 2023

Name: _____ Date: _____

Directions

Complete as many of the following problems as you can. You may use a calculator for help, but show your work! Partial credit will be awarded for good reasoning.

For the following problems, consider the following **parametrization**:
$x(t) = \frac{1-t^2}{1+t^2}$, $y(t) = \frac{2t}{1+t^2}$

1. what point (x and y coordinates) is output when...

   (a) $t = 0$?
   (b) $t = \frac{1}{3}$?
   (c) $t = \frac{1}{2}$?
   (d) $t = \frac{2}{3}$?
   (e) $t = 1$?

   [4 points]

2. What do you notice about the numerators and denominators of the coordinates you found so far? What is your conjecture about the relationship between these two rational numbers? [3 points]

3. What happens to $x(t)$ if t is negative? What is the relationship between $x(t)$ and $x(-t)$? [4 points]

4. What is the value of $x(t)x(t) + y(t)y(t)$? [5 points]

5. What open question(s) do you have about this relationship? [2 points]

This parametrization is special. Expect more questions about it in future challenges!