Challenge #0

THE MATH JESTER

Fall 2018

Consider the polynomial $g(x) = (x^3 + x)^2$.

1. What is the expanded form of $g(x)$?

   Solution. $g(x) = (x^3 + x)^2 = (x^3 + x)(x^3 + x) = x^6 + 2x^4 + x^2$.

2. Substitute (-x) for x. What is the result, i.e., what is g(-x)?

   Solution.

$$\begin{aligned} g(-x) &= ((-x)^3 + (-x))^2 \\ &= ((-(x^3) - x))^2 \\ &= ((-1)(x^3 + x))^2 \\ &= (-1)^2(x^3 + x)^2 \\ &= (x^3 + x)^2 \\ &= g(x) \end{aligned}$$

3. Now consider

$$h(x) = (x^3 + x)^{38}.$$

   What can you conclude about $h(x)$ and $h(-x)$? (There is no need to expand the expression!)

   Solution.

$$\begin{aligned} h(-x) &= ((-x)^3 + (-x))^3 8 \\ &= ((-(x^3) - x))^{38} \\ &= ((-1)(x^3 + x))^{38} \\ &= (-1)^{38}(x^3 + x)^{38} \\ &= (x^3 + x)^{38} \\ &= h(x) \end{aligned}$$

4. Is this interesting? (There are multiple correct answers. Use your judgment.)

   Solution. Answers may vary. Correct answers include "heck yea!", "Yup," "Yea, but not as much as abstract algebra," etc.

5. Now consider

$$j(x) = (x^3 + x)^{73}.$$

What is $j(-x)$? (Again, no need to expand. Just describe the relation between $j(-x)$ and $j(x)$.)

Solution.

$$\begin{aligned} j(-x) &= ((-x)^3 + (-x))^{73} \\ &= ((-(x^3) - x))^{73} \\ &= ((-1)(x^3 + x))^{73} \\ &= (-1)^{73}(x^3 + x)^{73} \\ &= -(x^3 + x)^{73} \\ &= -j(x) \end{aligned}$$